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Three Issues on the Kelly Criterion with Engineering Investment Projects

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Abstract : This paper is concerned with three issues associated with the applicability of the Kelly criterion to engineering investment projects instead of financial investment ones. It is believed that the criterion involves a high potential of being applied to the economic analysis of the engineering investment projects. From this perspective, this paper addresses three relevant issues: i) a violation of an existing popular stochastic dominance theorem, ii) the Kelly criterion with a time value of money, and iii) its optionality with a binomial lattice option pricing model. The presentation regarding the issues is primarily relied on as part of the author's research work, with the aid of previously existing literature.

Keyword : Binomial Lattice Option Pricing, Discounted Kelly Criterion Model, Markowitz Portfolio, Real Investment

1. Introduction

Generally speaking, a gambler and financial investor take two steps to implement his gambling/investing strategy as follows: in the first step, the gambler tries to find positive expectation betting opportunities and analogously, a financial investor also tries to find investments with excess risk–adjusted rates of return; in the second step, once these favorable opportunities have been identified, the gambler/financial investor must decide an optimal fraction of his capital to bet to maximize return on his betting or investing activity. To accomplish his goal, he may employ the Kelly criterion.

The Kelly criterion was developed and proposed by JohnKelly in 1956 which is much credited to Shannon's information theory [2]. Its main purpose is to decide an optimal bet size on each trial in a gambling game/financial investment so that a gambler/financial investor may maximize their expected value of the logarithm of capital, usually denoted by E[lnX] where a random variable of X represents an investment capital.

The criterion is therefore regarded as an optimal strategy to maximize a retrun on investment in gambling games and financial markets by taking advantange of an information edge there.

Thorp has been enthusiastically engaged in elaborating the Kelly criterion and as a researcher and practitioner in employing the Kelly criterion in both gambling games and stock investment[3]. Providing the illustrative examples based on his pronouncing experience, Thorp[4] revealed a relationship between the Kelly criterion and blackjack, sports betting and the stock market in depth. Thorp[5] also proved that the Kelly criterion generates the overwhelmingly better return on investment than any other investment strategy focusing on the theory of logarithmic utility as applied to financial investment portfolio selection and contrasting it with Markowitz mean-variance portfolio theory. In 1961 earlier than Thorp, Breiman[7] established the fundamental mathematical properties of the expected logarithm criterion in a rigorous fashion which is commensurate with the Kelly criterion. He proves them in a general discrete time setting with inter-temporally independent assets. There are many other researcher: Cover and Thomas[8], Friedman[9], MacLean and Ziemba[10], and Vince[11]. All of them have carried out the Kelly criterion in the fields of the gambling games and financial markets.

To attain the maximum expected value of logarithm of gamblers'/investors' wealth, his

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gambling or investing activity in the Kelly criterion is assumed to be repeatedly performed over a long period of time. It is one of the important assumption for the Kelly criterion, so much so that it is natural to take the time value of money in the Kelly criterion. Except for an anonymous short introduction to it[12], an existing literature survey explores that a formal research associated with it has yet to be done. The cash flows generated by the Kelly criterion follows a Bernoulli process as a binomial lattice model in a option pricing valuation does, so much so that Johnstone[13] and Zhang[14] proposed the applicability of the Kelly criterion to valuing financial options. Johnstone asserts that any investment position of assets, likely expressed in a binary term, may be duplicated by a traditional bet made at the odds that are based on risk-neutral rather than objective odds. Zhang discussed a relationship between the optimal geometric mean return of stock and its option value from the Kelly criterion perspective under the assumption of no sure betting strategy to win. In his work, he proved that the optimal geometric returns and optimal fractions of stocks and options depend on objective rather than risk-neutral probabilities, assuming that there exist no arbitrage opportunities in a financial market and one-step binomial lattice option pricing model appropriately represents investment. This proof is contradictory to Johnstone's.

In a practical aspect, Pounstone concretely describes in his book[3] that Thorp made a huge amount of wealth by employing and practicing the Kelly criterion in his stock investment and gambling games in Las Vegas as well. MacLean, Thorp, and Ziemba[15] claim that Warren E. Buffett, legendary value investor, has also employed the Kelly criterion as his investment strategy and been well known for achieving an extraordinary rate of return on his long-run investment in financial securities. As discussed above, all research has been focused on gambling games and financial investments. Except for one small tentative work which attempted to find out the possibility of applying the Kelly criterion to Engineering Economics[15], no explicit nor implicit research on the Kelly criterion with respect to an economic analysis of real investment projects has been done yet.

There may be various number of issues facing us en roue to successfully applying the Kelly criterion originally developed for the situation where gambling/investing capital can be infinitesimally divided to real investment environment where it is not easily allowed and relaxed. As an onset of work for the application, this paper is concerned with addressing three issues associated with its applicability to an economic appraisal of engineering investment projects. Firstly, the Kelly criterion violates a traditional stochastic dominance theorem and thus generates investment portfolios different from Markowitz portfolio theory[5]. Secondly, although the Kelly criterion holds its validity with the assumption that an investing activity is repeatedly carried out over a long period of time, it never considers the time value of money in its model. Thirdly, the criterion exactly follows a Bernoulli process which allows us to take a binomial lattice option pricing (BLOP) model into account with it. We will shortly discuss each of these issues in this paper based on part of the author's work with the aid of previously existing literature.

The paper is the revised version of the paper[16] and organized as follows: Section 2 will shortly present the basic concept of the Kelly criterion. Section 3 will discuss the stochastic dominance theorem from the Kelly criterion viewpoint and show a comparison result of Kelly criterion-based investment portfolios and Markowitz mean-variance-based portfolios. Section 4 will discuss the time value of money from a viewpoint of an economic analysis of real investment projects, which is completely ignored in a traditional Kelly criterion. Section 5 will demonstrate how to apply the Kelly criterion to an appraisal of the real investment projects under a BLOP environment. Finally, we will provide concluding remarks on the research.

2. Kelly Criterion Concept

The following assumptions are required to successfully implement the Kelly criterion[1, 4, 6]. They are:

- The expected rate of return must be greater than 0.
- It must be possible to reinvest the cash flows coming from investment.
- The existence of an independent relationship between the final cash flows must be held.
- The cash flows of investment must satisfy the conditions of a Bernoulli process.
- The investing activities must be carried out infinitely.
- It must be possible to dissemiate an investment capital in an infinitesimal size.
- The winning probability of p must be kept constant to investors.

After trials of investment with all the assumptions satisfied, an investors' wealth becomes

$$X_N = X_0(1+f)^W(1-f)^L \quad (1)$$

where X_0 is an investment capital at the outset, f is a portion of investment capital on hand to bet, and W and L represent a number of successes and failures, respectively. Dividing both sides of Equation (1) by X_0 and taking logarithm on them yield a logarithmic growth rate of investors' wealth, expressed by Equation (2).

$$G(f) = \ln\left(\frac{X_N}{X_0}\right) = \frac{W}{N}\ln(1+f) + \frac{L}{N}\ln(1-f) \quad (2)$$

And then its expected value is given by Equation (3).

$$g(f) = E\left\{\ln\left[\frac{X_N}{X_0}\right]^{1/N}\right\} = E\left\{\frac{W}{N}\ln(1+f) + \frac{L}{N}\ln(1-f)\right\}$$
$$= p\ln(1+f) + q\ln(1-f) \qquad (3)$$

where p and q are a probability of success and failure, respectively. More specifically stating, the term means the expected growth rate of return on investment attainable after investments are made for a long period of time.

By taking the first and second partial derivatives of the equation with respect to f and performing some mathematical work on it, we may obtain Equation (4) with its first derivative

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$$g'(f) = \frac{p}{1+f} - \frac{1}{1-f} = \frac{p-q-f}{(1+f)(1-f)} = 0 \qquad (4)$$

in which $f^* = p - q = 2p - 1$.

In Equation (4), if $p < \frac{1}{2}$, then $f^* < 0$, which implies that a short selling is suggested. Dur to no restrict on the condition of $p+q=1$, p in the equation probably takes no greater than zero. However, if $\frac{1}{2} \leq p \leq 1$, there may exist the non-trivial investment strategy of $0 < f^* < 1$.

And now we may have Equation (5) with the second derivative.

$$g''(f) = \frac{-p}{(1+f)^2} - \frac{q}{(1-f)^2} < 0 \qquad (5)$$

Equation (5) says that $g'(f)$ is monotone strictly decreasing on [0, 1). Since $g(0) = p - q = 2p - 1$ and $\underset{frarrow1^-}{\mathrm{Lim}}\, g(f) = -\infty$, $g(f)$ provides a unique maximum at $f = f^*$, where $g(f^*) = p\ln p + q\ln q + \ln 2 > 0$. In addition to the fact, $g(0) = 0$ and $\underset{frarrowq^-}{\mathrm{Lim}}\, g(f) = -\infty$, so there exists a unique number $f_c > 0$, where $0 < f^* < f_c < 1$, such that $g(f_c) = 0$ [6, 7]. Figure 1 shows an example where a return on investment becomes zero at f=23% at which and thereafter investors completely get ruined.

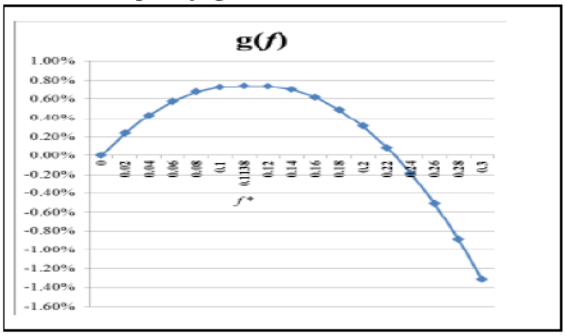

Figure 1. Relationship between g(f) and f

3. A Comparison of the Kelly Criterion and Markowitz Portfolio

In this section, we will present a summary of the comparison of the Kelly criterion-based portfolio(KCBP) with the Markowitz mean-variance portfolio(MMVBP) model. To the end, it is better to first discuss a proof that the former model generates a superior rate of return on investment to then latter one[5, 6]. We applied two models to two stocks, KAL and S-Oil, taken from KOSPI200 as shown in Table 1.

Hakansson demonstrates three intriguing and heretical findings in associatiion with the superiority of the KCBP to the MMVBP model in his paper [18]. The first one is that some of the portfolios made up with the KCBP model, being considered to generate much desirable result of return on investment than all the portfolios on the traditional efficient frotier line constructed with the MMVBP model, do not stay even adjacent to the line. The second one is that portfolios entailing investors' ruin in the long haul are some of the portfolios on the traditional Markowitz efficient frontier line. The last one is that the worst desirable portfolios with the KCBP model may produce the better rate of return on investment than most of the portfolios on the traditional efficient frontier lin in the long haul. We can find the numerical example regarding to these findings in Throp's paper[6]. Figure 2 shows the result of applying the Markowitz mean-variance portfolio theory to the KAL and S-Oil stocks. Table 1 shows a relevant data for the analysis.

Table 1. Return, and success/failure probability of KAL and S-Oil stocks

Stock	Sucs/Fail	Probability	Return
KAL	Success Failure	0.67 0.33	0.30 -0.59
S-Oil	Success Failure	0.56 0.44	0.42 -0.28

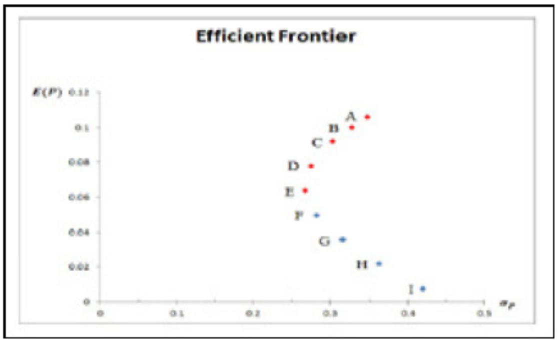

Figure 2. Markowitz's traditional efficient frontier

Figure 2. shows a combination of the mean and variance of the portfolios' return constructed under the MMVBP model. In the figure, point A, usually marked by (0,1), represents a portfolio in which one hundread percentage of investment capital on hand is distributed to S-Oil and nothing to KAL stocks. On the contrary, point I, marked by (1, 0), allocates investment capital in an opposite direction to point A. Here, points of ABCDE forms a traditional Markowitz efficient frontier. Taking into account of the Kelly criterion conditions and putting in for some mathematical work, portfolio B was chosen to dominate all other portfolios. In general, portfolios of this kind are determined regardless of where they are located. According to Hakansson as mentioned above, a portfolio staying on an outside of a traditional efficient frontier may be selected as the most desirable portfolio to invest[18]

With this advantage of the KCBP over the MMVBP model in mind, we made up two optimization models corresponding to the two portfolio models and applied them to all the stocks listed in KOSPI200, Their results are presented in Table 2 and 3.

Table 2. Result of the optimal KCBP model

No.	Stock Name	f_i^*	Return
1	HDW	75%	0.70%
2	SKC&C	11%	0.10%
3	HS	15%	0.10%
Expected Return with the KCBP			0.90%
Expected Return with KOSPI200			0.08%
Variance with the KCBP			0.30%
Variance with KOSPI200			0.20%

Table 3. Result of the optimal MMVBP model

No.	Stock Name	f_i^*	Return
1	HDW	56%	0.50%
2	SKC&C	24%	0.20%
3	HS	14%	0.10%
4	LGH	3%	0.02%
5	ORO	3%	0.01%
Expected Return with the MMVBP			0.81%
Expected Return with KOSPI200			0.08%
Variance with the MMVBP			0.17%
Variance with KOSPI200			0.17%

When investigating the results, one curious observation to note is that the KCBP model provides us with a less number of stocks to invest than the MMVBP one, by 2 in this case study. Many researchers like Thorp maintain that this observation can be usually found in many instances[5, 7, 10]. In terms of a portfolio management, it can be thus said that investors handle an investment portfolio with the KCBP model more effectively and conveniently than the MMVBP one. Warren Buffet once told that there were only several stocks in the NYSE market to pick up for an investment portfolio. His telling backs up the KCBP more preferably than the MMVBP model.

Besides, the KCBP model with a less number of stocks generates a rate of return on investment so much more highly than a more number of stocks with the MMVBP one which yielded greater risk than the former. Accordng to Hund's argument[19], the former yields twice as much risk as the latter. these findings are not specific to this short case, but common to most ones[4, 18]. When it comes to engineering investment projects economically evaluated and appraised with the KCBP model, a firm may select the best investment project which makes a profit much higher than an investment evaluation and appraisal practice widely in use in a current time.

4. Kelly Criterion with Time Value of Money

It is acceptable to ignore a time vlaue of money with the Kelly criterion with a gambling game and a financial investment project, which has been generally taken place and acknowledged in those fields. Its ignorance may, howenver, seriously make a distortion of a value of engineering investment projects. This distortion is expected to be incurred due to the fact that their planning horizon is in general longer than a year, and may be extended to 50 years or so in extreme cases. Taking the time value of money into consideration, we will briefly discuss how to incorporate it into the Kelly criterion on the basis of Kim's work[21].

Before the discussion, let's first provide the definiton of the variables which will be frequently employed in the following context. Let r stand for an interest rate per year and t: be the time at which investment payoffs are realized. The discussion will be proceeded supposing that a fraction of investment capital, f, is allocated to investment projects and the rest under a mattress. The remaining part put under the mattress earns nothing. Applying the concept of the time value of money, and the variables defined above and the assumption to Equation (3), we found a mathematical formula for the Kelly cirterion with the time value of money expressed by Equation (6). And the variance associated with this case is given by Equation (7) or (8).

$$DG_{\max}(Df^*) = p\ln\left\{\frac{p}{(1+f)^t}\right\} + q\ln\left\{\frac{q}{b}\right\} + \ln\{b+(1+r)^t\} \tag{6}$$

where,

$$Df^* = p - \frac{q}{b}(1+r)^t,$$

b: a payoff size with a successful investment.

$$Var(DGR) = pq\left[\ln\left\{1+\frac{b(Df)}{(1+r)^t}\right\} - \ln(1-Df)\right]^2 \quad (7)$$

$$= pq\left[\ln\left\{1+\frac{bp}{q(1+r)^t}\right\}\right]^2, \text{for } Df \neq Df^* \quad (8)$$

Looking at Figure 3, it is conceived that the higher f moves torward, the smaller and greater a growth rate and risk get, respectively. This fact causes the curves with the time value of money much steeper than those without it, especially as the planning horizon of engineering investment projects becomes longer and longer.

It is also found that all of a sudden risk gets started exponentially to go up toward a ceiling after $f = 25\%$. Howver, risk with the Kelly criterion without the time value of money begins to move up after $f = 30\%$. In this regard, a firm taking the

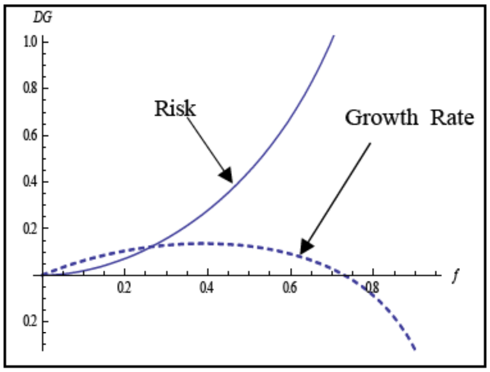

Figure 3. Profile of growth rate and risk with f

Kelly criterion with the time valuye of money is equipped with more caution about risk of investment projects and prepared for the undesriable situation happening in the future time to come. Meanwhile, the firms gets completely ruined in the region of $f > 73\%$, whereas $f < 73\%$ with a conventional Kelly criteria.

We took a short and real case study from the Korean professional baseball championship league in 2004. It was only the case that we could find real data in the Internet. In the year, Samsung Tigers and Nexen Heroes were advanced to the championship league. Analyzing the historical data on the games between two teams, we found that the proabilities that Samsung Tigers won, lost, and drawn over Nexen Heroes before the championship games were 64.5%, 34.5%, and 1%, respectively. These two teams had had 110 games before the time. Based on the analysis of the data, we projected that the odds in favor of Samsung Tigers and Nexen Heroes were 2.54 and 2.5, respectively. And the odd for a drawing game between two teams turned out to be 90.

We applied th Kelly criterion with the time value of money to this case study with a discount rate of 5% per year, r=5%/year assuming that bettors entered into the game: i) every 3 days, ii) every month, iii) every six month, iv) 1 year, v) 2 years, vi) 3 years, vii) 5 years, and viii) 10 years. The optimal fraction of f^* to bet was found from the Samsung Tiger's position.

Examining Figure 4, we can see that the longer a planning horizon of t and the bigger the optimal fraction of f^* are, the larger the amount of reduction in an expected growth rate and risk with the discounted Kelly criterion is. In addition, we also recognize that a betting ratio of f causing the bettors participated in the games to get ruined may have a positively proportional relationship to a planning horizon of t. Figure 5 presents the pattern of Df^* behavior when r and t concurrently change. We can recognize in the figure that Df^* begins steeply to decline when a discount rate of r and a planning horizon of t get into 7% and 10 years, respectively.

In summary, due to a significant effecto of a planning horizon of t and an interest rate of r on the Kelly criterion performance, it is better to take into consideration of their effect on an economic evaluation and appraisal of engineering investment projects, especially whose planning horizon is long enough.

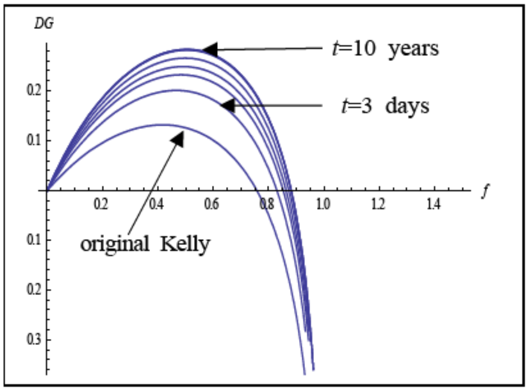

Figure 4. Growth rate and betting ratio with different time horizon

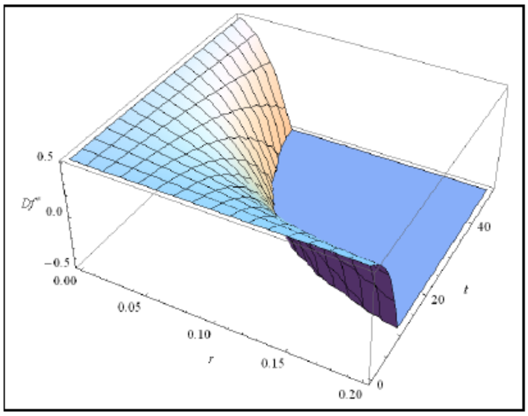

Figure 5. Df* with different time horizon and annualized interest rate of r

5. Optionality with a Binomial Lattice Model

In this section, we will discuss about the Kelly criterion in relation to engineering investment projects with a binomial lattice option pricing(BLOP) model. Johnstone[13] clarified that there existed a relationship between the long and short positions in the underlying asset, and traditioanl Kelly bets might be made at risk neutral betting odds. Also, Zhang[14] and Zhu[22] proved a relationship between the udnerlying asset for option and the Kelly criterion supposing that there was no sure betting strategy to win.

Applying the conventional Kelly criterion to a BLOP model results in the logarithmic utility function exprerssed in $U(X_N)_{\max}$ as seen in Equation (9)[13, 22].

$$U(X_N)_{\max} = p\ln\left\{p\frac{(1+r_f)^t}{p'}\right\} + (1-p)\ln\left\{(1-p)\frac{(1+r_f)^t}{(1-p')}\right\} + \ln X_0 \quad (9)$$

where,

p: an objective probability,
p': a risk-neutral probability,
X_0 : an original investment capital,
r_f : a risk-default interest rate.

It is an eccentric fact that we do not find out the optimal betting ratio, f^*, in this example. The reason for the fact happening is that looking at Equation (9), we can notice that the optiaml fraction of f^* is replaced with an objective probability of p measured with real data collected in a market. Applying the optionality concept with the Kelly criteron under a BLOP environment to an engineering replacement problem in which each of projects has a mutually exclusive relationship with another and a unequal live, as a result we obtained Figure 6 showing an optimal betting fraction and its corresponding final values when the following projects would be replaced with one selected at the first time over a long period of time. This figure was depicted with a uncertain risk-default interest rate only. This application was implemeted with the data in Table 5.

A risk-default interest rate was assumed to be uncetain with its expected value of 6% and standard deviation of 1%. In Figure 6, we can conceive that project B is prefered to project A prior to an optimal betting fraction of 89%. The opposite result shows up thereafter. The comparison of the results of three different scenarios are presented in Table 5; (1) In Scenario I, it is assumed that a risk-default interest rate is absolutely certain, (ii) In Scenario II, it is assumed that a risk-default interest rate is uncetain and thus a traditional BLOP model is applicale to the example, and (iii) In Scenario III, the Kelly criterion with a BLOP model is applied to the example where there exists uncertainty in a risk-default interest rate.

Table 4. Cash flows of Project A and B

Year	Project	
	A	B
0	-150	-148
1	7.5	16.5
2	7.5	16.5
3	187.5	16.5
4		16.5
5		166.5

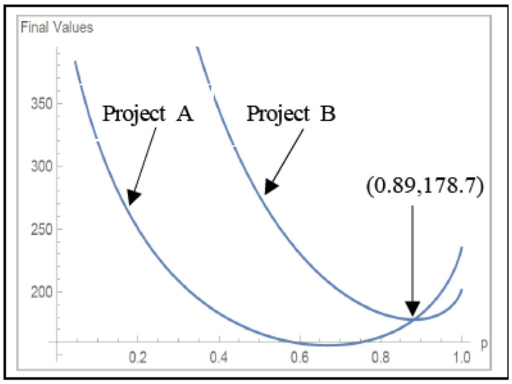

Figure 6. Optimal fraction and its corresponding final values when projects will be replaced with one selected at the first time over an long period of time.

Table 5. Comparison of the results

Scenario	Project		Final Decision
	A	B	
Fixed	132	133	B
Option	139	139	A
Kelly	167	276	B

Table 5 presents the results of the scenarios. Note that only the project value for the third scenario is presented with p=50%. In this table, we can observe that two scenarios but the second one generate the identical final decision. In addition, the project value becomes larger and larger from the first scenario to the third one. Care should be exercised that this observation is subject to this short example at this moment.

6. Concluding Remarks

We have presented a discussion about three issues, which were presumably considered to play an innportant role in the application of the Kelly criterion to engineering investment projects. Even if the discussion missed more important issues and is incomplete, it is believed that a tentative onset attempt of a sort is worth pursuing. The first issue causes engineering investment project analysts to reconsider traditional stochastic dominance theorems frequently employed to select the most desirable project A return on gambling games and financial investments is enough to safitisfy a myopically set-up goal usually specified within a year or so at a maximum in these days. However, an absolutely different story must go on with the engineering investment proejcts whose planning horison as mentioned above may be extended to 40-50 years or so in an extreme case. It is, therefore, unbelievable to ignore the time value of money especially when applying the Kelly criterion to evaluate and appraise the engineerng investment projects. In this respect, the second issue msut be kept in mind in an economic evaluation of the engineering investment projects. And this research raised the third issue of the applicability of the Kelly criterion under an BLOP environment to the projects. Each of the issues in isolation even plays an important role in their economic analysis. Moreover, it is supposed that they all put in together will have a significant effect on the analysis.

As the last but not least comment on the research and a future research direction, it is asked to find out a way to resolve what and how to define an investment capital in association with the domain of the Kelly criterion. It is a trivial issue in financial projects, but it is sophiscated in engineirng investment projects even if it seemingly looks easy and simple to be resolved. Based on our research experience and literature survey, it can be mentioned that the issue has been hardly dealt with and henceforth waiting for being defintely defined in a near future time to come. Sooner or later, we will be planning to take the issue as our future research topic as well and expect to derive conceptually and mathematically crystallized outcomes.

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REFERENCES

1. J. L. Kelly, “A new interpretation of information rate” The Bell System Technology Journal, vol. 35, no. pp. 917–926(1956).
2. C. E. Shannon, “A mathematical theory of communication” The Bell System Technology Journal, vol. 27, pp. 379-423(1948).
3. W. Poundstone, Fortune’s Formula: The Untold Story of the Scientific Betting System that beat the casino and Wall Street, New York, NY, Hill and Wang(2006).
4. E. O. Thorp, “The Kelly criterion in blackjack, sports betting, and the stock market” in Proc. of 10th Int. Conf. on gambling and risk taking, http://www.bjmath.com/b(1997).
5. E. O. Thorp, Portfolio choice and the Kelly criterion” in Bus. and Eco. Stat. Sec. of Proc. of the Amer. Stat Ass., pp. 215-224(1971).
6. E. O. Thorp, “Optimal gambling systems for favorable games” Rev. of the Int. Stat. Inst. vol. 3, No. 3, pp. 272-293(1969).
7. L. Brieman, “Optimal gambling systems for favorite games” in Proc. of the 4th Berkeley Sym. on Math. Stat. and Pro., Vol. I, Berkeley and LA, CA, pp. 65-78(1961).
8. T. Cover and J. A. Thomas, Elements of Information Theory, Wiley Series in Telecommunications, NY(1991).
9. J. H. Friedman, “Understanding and applying the Kelly crtierion”, Proc. 3rd Conf on Gambling and Risk Taking, Las Vegas, Nevada, pp. 128-139(1976).
10. L. C. MacLean and W. T. Ziemba, “Capital growth: Theory and Practice”, Handbook of Asset and Liability Management, Vol. 1, pp. 429-472(2008).
11. R. Vince, Protfolio Management Formula: Mathematical Trading Methods for the Futures, Options, and Stock Markets, NY, John Wiley(1990).
12. Australiasportsbetting.com ”Kelly criterion-Part 3b- Accounting for the time value of money”, http://www.aussportsbetting.com/2010/07/21/kelly-criterion-accounting-for-the-time-value-of-monet/, (2010).
13. D. Johnstone, “Investments as bets in the binomial asset pricing model” in Forecasting Expected Returns in the Financial Markets edited by Satchell, S., Amsterdam, Netherland, Academic Press(2007).
14. G. Zhang, “Optimal geometric mean returns of stocks and their options” Int. J. of Stochastic Ana. pp. 1-8 (2012).
15. G. T. Kim, “On the applicability of the Kelly criterion to Engineerign Economics”, 2008 IEEE International Conference on IEEM, Singapore, pp. 267-272(2008).
16. G. T. Kim, “Some thoughts on the Kelly criterion associated with a real investment perspective”, 2017 IEEE International Conference on IEEM, Singapore, pp. 1641-1645(2017).
17. L. C. MacLean, E. O. Thorp, and W. T. Ziemba, “Good and bad properties of the Kelly criterion”, https://www.stat.berkeley.edu/~aldous/157/Papers/Good_Bad_Kelly.pdf
18. N. H. Hakansson, “Capital growth and the mean-variance investment policies” J. of Fin. And Quant. Anal. vol. 6, pp. 1-21(1981).
19. G. T. Kim, “A comparison of the Kelly criterion and the Markowitz’s mean-variance model to portfolio selection with KOSPI200’, Ind. Eng. & Man. Sys. in press.

20. B. Hund, "Growth optimal investment strategy efficacy: an application on long run Austrian equity data" Inv. Mana. & Fin. Inn., pp. 8-22, January(2005).
21. G. T. Kim, "The Kelly criterion with the time value of money" submitted to IJIE.
22. Q. Zhu, "Investment system specific option pricing intervals in incomplete markets" in Proc. of the 2007 Amer. Control Conf., New York, NY, pp. 402-407(2007).
23. G. T. Kim "A replacement analysis of mutually exclusive projects of unequal lives with the Kelly real options criterion" under review.