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Differential Equations on Riemannian Warped Product Manifolds

朝鮮大學校 教育大學院

數學敎育專攻

尹 慧 梨

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리만 휜 다양체 위의 미분방정식

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본 논문에서는 N이 컴팩트 리만다양체이고, a가 양의 상수인 $M=[a,\infty)\times_f N$위의 리만거리를 특별한 스칼라 곡률과 함께 구성하기 위하여 휜 곱을 사용하는 방법에 대해 논의하기로 한다. N을 M의 경계로 사용함으로써 M위에 휜 곱을 적용할 수 있는데, 만약 엽다양체(fiber manifold) N이 (A), (B) 또는 (C)부류에 속한다면 상수가 아닌 휜 함수를 사용하여 M위의 리만 휜 거리에서 상수인 스칼라곡률을 얻을 수 있다. 이를 위해 기하학적인 문제를 미분방정식의 해를 구하는 해석학적인 문제로 바꾸어, 그 미분방정식의 해를 구하여 기하학적 성질을 얻었다.

Differential Equations on Riemannian Warped Product Manifolds

Ⅰ. INTRODUCTION

In a recent study [13,14], Leung studied the problem of scalar curvature functions on Riemannian warped product manifolds and obtained partial results about the existence and nonexistence of Riemannian warped metric with some prescribed scalar curvature function. In this paper, we study also the existence and nonexistence of special Riemannian warped product manifolds.

By the results of Kazdan and Warner [10-12], if N is a compact Riemannian n-manifold without boundary, $n \geq 3$, then N belong to one of following three categories:

(A) A smooth function on N is the scalar curvature of some Reimannian metric on N if and only if the function is negative somewhere.

(B) A smooth function on N is the scalar curvature of some Reimannian metric on N if and only if the function is either identically zero or strictly negative somewhere.

(C) Any smooth function on N is the scalar curvature of some Reimannian metric on N.

This completely answers the question of which smooth functions are scalar curvatures of Riemannian metrics on a compact manifold N.

In [10-12], Kazdan and Warner also showed that there exists some obstruction of a Riemannian metric with positive scalar curvature (or zero scalar curvature) on a compact manifold.

For noncompact Riemannian manifolds, many important works have been done on the question of how to determine which smooth functions are scalar curvature of complete Riemannian metrics on an open manifold. Results of Gromov and Lawson[9] show that some open manifolds cannot carry complete Riemannian of positive scalar curvature, for example, weakly enlargeable manifolds. Furthermore, they show that some open manifolds cannot even admit complete Riemannian metrics with scalar curvatures uniformly positive outside a compact set and with Ricci curvatures bounded [9,15,p.322].

On the other hand, it is well known that each open manifold of dimension bigger than 2 admits a complete Riemannian metric of constant negative scalar curvature [7]. It follows from the results of Aviles and McOwen [3] that any bounded negative function on an open manifold of dimension bigger than 2 is the scalar curvature of a complete Riemannian metric.

In [13,14], Leung considered the scalar curvature of some Riemannian warped product and its conformal deformation of warped product metric. And also in [8], Ehrlich et al. considered the existence of a nonconstant warping function on a Lorentzian warped product manifold such that the resulting warped product metric produces the constant scalar curvature when the fiber manifold has the constant scalar curvature.

In this paper, compared the Lorentzian results in [8], the Riemannian version is considered. That is, when N is a compact Riemannian manifold, we discuss the method of using warped products to construct Riemannian metrics on $M = [a, \infty) \times_f N$ with specific scalar curvatures, where a is a positive constant. By making use of the boundary, we can construct warped products at the end of M. It is shown that if the fiber manifold N belong to class (A), (B) or (C), then, using a nonconstant warping function, we obtain a Riemannian warped product metric on M admitting a constant scalar curvature.

Ⅱ. Preliminaries on a warped product manifold

In this section, we briefly recall some results on warped product manifolds. Complete details may be found in [4], [6], or [16].

DEFINITION 2.1 (Riemannian connection) A Riemannian connection is a bilinear map D of $T(M)\times\Gamma_1(M)$ into $T(M)$ such that

(a) $D(X_p, Y) = D_{X_p}(Y) \in T(M)$ where $X_p \in T_P(M)$.

(b) If f is a differentiable function,

$D_{X_p}(fY) = X_P(f)Y + f(p)D_{X_p}(Y)$.

(c) If X and Y belong to Γ_1, X is of class C^r and Y of class C^{r+1}, then $D_X Y$ is of class C^r.

(d) D is torsion free, $i.e.$, $T(X,Y) = D_X Y - D_Y X - [X,Y] = 0$.

(e) D is compatible with metric, $i.e.$, $D_k g_{ij} = 0$ for all i,j,k ([2]).

DEFINITION 2.2 (Curvature) The curvature of the connection is the 2-form with value in $Hom(\Gamma_1, \Gamma_1)$ defined by:

$R(X,Y) = D_X D_Y - D_Y D_X - D_{[X,Y]}$.

one verifies that $R(X,Y)Z$ at p depends only upon the values of X, Y, and Z at p ([2]).

In a local chart, denote by R_{kij}^{l} the lth component of $R(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j})\frac{\partial}{\partial x^k} = \Sigma_{l=1}^{n} R_{kij}^{l} \frac{\partial}{\partial x^l}$ ([1]) which is called the curvature tensor, and

$$R_{kij}^{l} Z^k = (\nabla_i \nabla_j Z)^l - (\nabla_j \nabla_i Z)^l$$

It follows that

$$R_{kij}^{l} = \partial_i \Gamma_{jk}^{l} - \partial_j \Gamma_{ik}^{l} + \Gamma_{im}^{l} \Gamma_{jk}^{m} - \Gamma_{jm}^{l} \Gamma_{ik}^{m}.$$

DEFINITION 2.3 From the curvature tensor, only one nonzero tensor (or its negative) is obtained by contraction. It is called the *Ricci tensor*. Its components are $R_{ij} = \Sigma_{k=1}^{n} R_{ikj}^{k}$. The Ricci tensor is symmetric and its contraction $R = \Sigma_{ij=1}^{n} R_{ij} g^{ij}$ is called the *scalar curvature* ([2]).

DEFINITION 2.4 (Laplacian) The Laplacian in a local chart can be written as follows:

$$\begin{aligned} \triangle \varphi &= \nabla_i (g^{ij} \nabla_j \varphi) \\ &= \partial_i (g^{ij} \partial_i \varphi) + g^{kj} \partial_j \varphi \Gamma_{ik}^{i} \end{aligned}$$

$$\triangle \varphi = |g|^{-\frac{1}{2}} \partial_i [g^{ij} \sqrt{|g|} \partial_j \varphi]$$

because $\Gamma^i_{ik} = \partial_k \log\sqrt{|g|}$ ([2,17]).

On a Riemannian product manifold $B\times F$, let π and σ be the projections of $B\times F$ onto B and F, repectively, and let $f>0$ be a smooth function on B.

DEFINITION 2.5 The warped product manifold $M=B\times_f F$ is the product manifold $M=B\times F$ furnished with metric tensor

$$g = \pi^*(g_B) + (f\circ\pi)^2\sigma^*(g_F),$$

where g_B and g_F are metric tensors of B and F, respectively. In other words, If v is tangent to M at (p,q), then

$$g(v,v) = g_B(d\pi(v),d\pi(v)) + f^2(P)\, g_F(d\sigma(v),d\sigma(v)).$$

Here B is called the base of M and F the fiber. We denote the metric g by < , >. In view of Remark 2.6-(1) and Lemma 2.7, we may also denote the metric g_B by < , >. The metric g_F will be denote by (,).

REMARK 2.6 Some well known elementary properties of the warped product manifold $M=B\times_f F$ are as follows.

(1) For each $q\in F$, the map $\pi|_{\sigma^{-1}(q)=B\times q}$ is an isometry onto B

(2) For each $p\in B$, the map $\sigma|_{\pi^{-1}(q)=p\times F}$ is a positive homothetic map onto F with homothetic factor $1/f(p)$.

(3) For each $(p,q) \in M$, the horizontal leaf $B \times q$ and the vertical fiber $p \times F$ are orthogonal at (p,q).

(4) The horizontal leaf $\sigma^{-1}(q) = B \times q$ is a totally umbilic submanifold of M.

(5) If ϕ is an isometry of F, then $1 \times \phi$ is an isometry of M. And if ψ is an isometry of B such that $f = f \circ \psi$, then $\psi \times 1$ is an isometry of M.

Recall that vector tangent to leaves are called horizontal and vectors tangent to fibers are called vertical. From now on, we will often use a natural identification $T_{(p,q)}(B \times_f F) \cong T_{(p,q)}(B \times F) \cong T_p B \times T_q F$. The decomposition of vector field on B, we define $\overline{X}$ at (p,q) by setting $\overline{X}(p,q) = (X_p, 0_q)$.`Then $\overline{X}$ is π-related to X and σ-related to the zero vector field on F. Similarly, if Y is a vector field on F, $\overline{Y}$ is defined by $\overline{Y}(p,q) = (0_p, Y_q)$.

LEMMA 2.7 If h has a smooth function on B, then the gradient of the lift $h \circ \pi$ of h to M of gradient of h on B.

PROOF. See Lemma 7.34 in [16].

In view of Lemma 2.3, we simplify the notation by writing h for $h \circ \pi$ and $grad(h)$ for $grad(h \circ \pi)$. For a covariant tensor A on B, its lift $\overline{A}$ to M is just its pullback $\pi^*(A)$ under the projection $\pi : M \to B$. That is, if A is a $(1,s)$-tensor, and if $v_1, \cdots v'_s \in T_{(p,q)}M$, then $\overline{A}(v_1, \cdots, v_s) = A(d\pi(v_1)), \cdots, d\pi(v_s))$ $\in T_p(B)$. Hence if v_k is vertical, then $\overline{A} = 0$` on B. For example, if f is a smooth function on B, the lift to M of the Hessian of f is also denoted by

H^f. This agrees with the Hessian of the lift $f \circ \pi$ generally only on horizontal vectors. For detailed computations, see Lemma 5.1 in [5].

Now we recall the formula for the Ricci curvature tensor Ric of the warped product manifold $M = B \times_f F$. We write Ric^B for the pullback by π of the Ricci curvature of B and similarly for Ric^F.

LEMMA 2.8 On a warped product manifold $M = B \times_f F$ with $n = \dim F > 1$, let X, Y be horizontal and V, W vertical. Then

(1) $Ric(X, Y) = Ric^B(X, Y) - \frac{n}{f} H^f(X, Y)$

(2) $Ric(X, V) = 0$

(3) $Ric(V, W) = Ric^F(V, W) - < V, W > f^{\#}$,

where $f^{\#} = \frac{\triangle f}{f} + (n-1)\frac{< gradf, gradf >}{f^2}$, and $\triangle f$ is the Laplacian on B.

PROOF. See Corollary 7.43 in [17].

On the given warped product manifold $M = B \times_f F$, we also write S^B for the pullback by π of the scalar curvature S_B of B and similarly for S^F. From now on, we denote $grad(f)$ by ∇f.

COROLLARY 2.9 If S is the scalar curvature of $M = B \times_f F$ with $n = \dim F > 1$, then

$$S = S^B + \frac{S^F}{f^2} - 2n\frac{\triangle f}{f} - n(n-1)\frac{< \nabla f, \nabla f >}{f^2}, \qquad (2.1)$$

where $\triangle$ is the Laplacian on B.

PROOF. For each $(p,q) \in M = B \times_f F$, let $\{e_i\}$ be an orthonormal basis for T_pB. Then by the natural isomorphism $\{\overline{e_i} = (e_i, 0)\}$ is an orthonormal set in $T_{(p,q)}M$. We can choose $\{d_j\}$ on T_qF such that $\{\overline{e_i}, \overline{d_j}\}$ forms an orthonormal basis for $T_{(p,q)}M$. Then

$$1 = < \overline{d_j}, \overline{d_j} > f(p)^2 (d_j, d_j) = (f(p)d_j, f(p)d_j),$$

which implies that $\{f(p)d_j\}$ forms an orthonormal basis for T_qF.
By Lemma 2.8 (1) and (3), for each i and j,

$$Ric(\overline{e_i}, \overline{e_i}) = Ric^B(\overline{e_i}, \overline{e_i}) - \Sigma_i \frac{n}{f} H^f(\overline{e_i}, \overline{e_i}),$$

and

$$Ric(\overline{d_j}, \overline{d_j}) = Ric^F(\overline{d_j}, \overline{d_j}) - f^2(d_j, d_j)\left(\frac{\triangle f}{f} + (n-1)\frac{< \nabla, f, \nabla f >}{f^2}\right)$$

Hence for $\varepsilon_\alpha = \mathrm{g}\,(e_\alpha, e_\alpha)$,

$$S(p,q) = \Sigma_\alpha \varepsilon_\alpha R_{\alpha\alpha}$$
$$= \Sigma_i \varepsilon_i Ric(\overline{e_i}, \overline{e_i}) + \Sigma_j \varepsilon_j Ric(\overline{d_j}, \overline{d_j})$$
$$= S^B(p,q) + \frac{S^F}{f^2} - 2n\frac{\triangle f}{f} - n(n-1)\frac{< \nabla f, \nabla f >}{f^2},$$

which is a nonlinear partial differential equation on $B \times q$ for each $q \in F$.

Now we may pose the following question: if $S_F(q) \equiv c$ (constant) on F, can we find a warping function $f > 0$ on B such that the warped metric g has constant scalar curvature $S(p,q) = k$ on $M = B \times_f F$? If $S(p,q) \equiv k$ for all $(p,q) \in M$, then equation (2.1) is the pullback by π of the following equation:

$$k = S_B(p) + \frac{c}{f^2} - 2n\frac{\triangle f}{f} - n(n-1)\frac{< \nabla f, \nabla f >}{f^2},$$

or equivalently,

$$\triangle f + \frac{1}{2n}(k - S_B)f - \frac{c}{2nf} + \frac{n-1}{2}\frac{< \nabla f, \nabla f >}{f} = 0. \qquad (2.2)$$

Ⅲ. Main Results

In this section we restrict our result to the case that $B=(a,b)$ is an open connected subset of R with the positive definite metric $+dt^2$ and $-\infty \le a < b \le +\infty$. Recalling that $\triangle f = +f''(t)$ and $< \nabla f, \nabla f > = +(f'(t))^2$, and making the change of variable $f(t) = \sqrt{v(t)}$, we have following equation from equation (2.2),

$$v''(t) + \frac{(n-3)}{4}\frac{|v'(t)|^2}{v(t)} + \frac{k}{n}v(t) - \frac{c}{n} = 0 \qquad (3.1)$$

where we assume that F is a Riemannian manifold with constant scalar curvature c and $\dim F = n > 1$ (cf. equation (2.16) in [4,p78]).

Now we consider the following problem:

Problem Ⅰ: Given a fiber F with constant scalar curvature c, can we find a warping function $f>0$ on $B=(a,b)$ such that for any real number k, the warped metric g admits k as the constant scalar curvature on $M=(a,b)\times_f F$?

We consider several cases according to the dimension of F and the value of the given c.

THEOREM 3.1 If $\dim F = n = 3$, then for any real number k the following warping function $v(t)$ produced constant scalar curvature k on (M, g):

i) $k>0, \quad v(t)=c_1\sin\left(\sqrt{\frac{k}{3}}\,t\right)+c_2\cos\left(\sqrt{\frac{k}{3}}\,t\right)+\frac{c}{k},$

ii) $k=0, \quad v(t)=\frac{c}{6}t^2+c_1t+c_2,$

iii) $k<0, \quad v(t)=c_1\exp\left(\sqrt{-\frac{k}{3}}\,t\right)+c_2\exp\left(-\sqrt{-\frac{k}{3}}\,t\right)+\frac{c}{k},$

where c_1 and c_2 are suitable constants chosen (if possible) so that $v(t)$ is positive on $B=(a,b)$.

PROOF. If $n=3$, then we have a simple differential equation.

$$v''(t)+\frac{k}{3}v(t)-\frac{c}{3}=0$$

Putting $h(t)=-\frac{k}{3}v(t)+\frac{c}{3}$, it follows that $h''(t)+\frac{k}{3}h(t)=0$. Hence, according to sign oh k, the above solutions follow directly from elementary method for ordinary differential equations.

REMARK 3.2 The difficulty in applying Theorem 3.1 is simply to insure that $c_{1,}c_2$ may be chosen, depending on c, k, and the interval $B=(a,b)$ such that $v(t)$ is positive for all $t\in(a,b)$. The strongest statement that may be made independent of choice of (a,b) is the following.

COROLLARY 3.3. For $\dim F=3$ and (a,b) arbitrary.

i) for $k>0$, Problem I may be solved affirmatively for all $c>0$,

ii) for $k=0$, Problem I may be solved affirmatively for all $c\geq 0$

iii) for $k<0$, Problem I may be solved affirmatively for all c.

REMARK 3.4 If $k=0$, $c<0$ and $B=(a,b)=(-\infty,+\infty)$, then no value of c_1,c_2 may be chosen which will produce a warping function positive on all of $(-\infty,+\infty)$. Similarly, if $k>0, c\leq 0$ and $B=(a,b)=(-\infty,+\infty)$, then no values of c_1,c_2 will produce $v(t)>0$ on all of $(-\infty,+\infty)$.

THEOREM 3.5 If $\dim F=n\neq 3$ and $c=0$, then for any real number k the warping function $v(t)$ produces constant scalar curvature k on (M, g):

i) $k>0$, $v(t)=\left(c_1\cos\left(\sqrt{\frac{(n+1)k}{4n}}\,t\right)+c_2\left(\sin\left(\sqrt{\frac{(n+1)k}{4n}}\,t\right)\right)\right)^{\frac{4}{n+1}}$

ii) $k=0$, $v(t)=(c_1t+c_2)^{\frac{4}{n+1}}$,

iii) $k<0$, $v(t)=\left(c_1\exp\left(\sqrt{-\frac{(n+1)k}{4n}}\,t\right)+c_2\exp\left(-\sqrt{-\frac{(n+1)k}{4n}}\,t\right)\right)^{\frac{4}{n+1}}$,

where c_1 and c_2 are suitable constants chosen (if possible) so that $v(t)$ is positive.

PROOF. In this case, equation (3.1) is changed into the simple form,

$$\frac{v''(t)}{v(t)}+\frac{(n-3)}{4}\frac{v'(t)^2}{v(t)^2}+\frac{k}{n}=0.$$

Putting $v(t)=\omega(t)^{\frac{4}{n+1}}$, then $\omega(t)$ satisfies the equations,

$$v'(t)=\frac{4}{n+1}\omega(t)^{\frac{4}{n+1}-1}\omega'(t)$$

and

$$v''(t)=\frac{-4(n-3)}{(n+1)^2}\omega^{\frac{4}{n+1}-2}\omega'(t)^2+\frac{4}{n+1}\omega^{\frac{4}{n+1}-1}\omega''(t).$$

Hence $\omega''(t)=\frac{-(n+1)}{4n}k\omega(t)$ and our solutions follow.

REMARK 3.6 (1) If $k<0$ and (a,b) is arbitrary, taking $c_1=c_2=1$ in Theorem 3.5 provides an affirmative solution to Problem Ⅰ.

(2) If $k=0$ and $B=(-\infty,+\infty)$, only a constant warping function $v(t)$ with $c_1=0, c_2>0$ will satisfy $v(t)>0$ on all of B.

(3) If $k>0$ and $B=(-\infty,+\infty)$, then ⅰ) reveals that Problem Ⅰ may not be solved on all of B. In the case that B is a finite interval, evidently ⅰ) reveals that a positive warping function $v(t)$ may be constructed.

THEOREM 3.7 If $\dim F=n\neq 1,3$ and $c\neq 0$, then for any real number k the warping function $v(t)$ produces constant scalar curvature k on (M, g):

i) $k>0,\ v(t)=\dfrac{n+1}{n-1}\dfrac{c}{k}[\tan^2(\pm\sqrt{\dfrac{k}{n(n+1)}}\,t+c_1)-1]^{-1},$

ii) $k=0,\ v(t)=\dfrac{c}{n(n-1)}t^2+c_1t+\dfrac{n(n-1)}{4c}c_1^2,$

iii) $k<0,\ v(t)=\left[c_1\exp\left(-\sqrt{\dfrac{-k}{n(n+1)}}\,t\right)+\dfrac{n+1}{n-1}\dfrac{c}{4kc_1}\exp\left(\sqrt{\dfrac{-k}{n(n+1)}}\,t\right)\right]^2$

where c_1 is a suitable constant chosen (if possible) so that $v(t)$ is positive.

PROOF. Suppose $v(t)$ is a solution of equation (3.1). If $v(t)$ is a constant, then $v(t)=\frac{c}{k}$, which is defined only when $ck>0$. If $v(t)$ is nonconstant, putting $v(t)=\omega(t)^{\frac{4}{n+1}}$, then $\omega(t)$ satisfies the equations,

$$v'(t)=\frac{4}{n+1}\omega(t)^{\frac{4}{n+1}-1}\omega'(t)$$

and

$$v''(t)=\frac{-4(n-3)}{(n+1)^2}\omega^{\frac{4}{n+1}-2}\omega'(t)^2+\frac{4}{n+1}\omega^{\frac{4}{n+1}-1}\omega''(t).$$

Hence

$$\omega''(t)+\frac{n+1}{4n}k\omega(t)-\frac{n+1}{4n}c\omega^{1-\frac{4}{n+1}}=0.$$

Putting $\dfrac{d\omega(t)}{dt} = y$ and $\dfrac{dy}{dt} = \omega''(t)$,

$$\frac{d\omega}{dy} = \frac{y}{-\dfrac{k(n+1)}{4n}\omega + \dfrac{c(n+1)}{4n}\omega^{1-\frac{4}{n+1}}}$$

$$y^2 = \frac{n+1}{4n}\omega^2\left(-k + \frac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}\right)$$

and

$$\frac{d\omega}{\omega\sqrt{-k + \dfrac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}}} = \pm\sqrt{\frac{n+1}{4n}}\,dt$$

Here we have three following cases:

$$\begin{aligned}\int \frac{d\omega}{\omega\sqrt{-k + \dfrac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}}} &= -\frac{n+1}{2\sqrt{k}}\tan^{-1}\left(\frac{\sqrt{-k + \dfrac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}}}{\sqrt{k}}\right), && k > 0 \\ &= \sqrt{\frac{(n+1)(n-1)}{4c}}\,\omega^{\frac{2}{n+1}}, && k = 0 \\ &= -\frac{n+1}{4\sqrt{-k}}\log\frac{\sqrt{-k + \dfrac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}} - \sqrt{-k}}{\sqrt{-k + \dfrac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}} + \sqrt{-k}}, && k < 0\end{aligned}$$

Hence our results follow easily. For example, if $k<0$, then

$$\sqrt{-k+\frac{n+1}{n-1}c\omega^{-\frac{4}{n+1}}}=\frac{\sqrt{-k}\left(1+\tilde{c}_1\exp\left(\sqrt{\frac{-4k}{n(n+1)}}\,t\right)\right)}{1-\tilde{c}_1\exp\left(\sqrt{\frac{-4k}{n(n+1)}}\,t\right)}$$

for some constant $\tilde{c}_1$. Thus

$$\begin{aligned} v(t) &= \omega(t)^{\frac{4}{n+1}} \\ &= \frac{n+1}{n-1}\frac{c}{(-4k\tilde{c}_1)}\left[\exp\left(-\sqrt{\frac{-k}{n(n+1)}}\,t\right)-\tilde{c}_1\exp\left(\sqrt{\frac{-k}{n(n+1)}}\,t\right)\right]^2, \end{aligned}$$

which implies the first, replacing $\tilde{c}_1=\frac{n+1}{n-1}\frac{c}{(-4kc_1^2)}$

REMARK 3.8 (1) If $k>0$ and $B=(-\infty,+\infty)$, then i) of Theorem 3.7 reveals that no warping function $v(t)$ may be found which is positive on all of B.

(2) If $k<0$, then iii) of Theorem 3.7 reveals that Problem I may be solved affirmatively for any B provided that $c>0$.

(3) If $k=0$, then ii) of Theorem 3.7 reveals that problem I may be solved affirmatively for a finite interval B provided that $c>0$.

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논문제목	한글 : 리만 휜 다양체 위의 미분방정식 영문 : Differential Equations on Riemannian Warped Product Manifolds				

본인이 저작한 위의 저작물에 대하여 다음과 같은 조건 아래 조선대학교가 저작물을 이용할 수 있도록 허락하고 동의합니다.

\- 다 음 -

1. 저작물의 DB구축 및 인터넷을 포함한 정보통신망에의 공개를 위한 저작물의 복제, 기억장치에의 저장, 전송 등을 허락함
2. 위의 목적을 위하여 필요한 범위 내에서의 편집・형식상의 변경을 허락함. 다만, 저작물의 내용변경은 금지함.
3. 배포・전송된 저작물의 영리적 목적을 위한 복제, 저장, 전송 등은 금지함.
4. 저작물에 대한 이용기간은 5년으로 하고, 기간종료 3개월 이내에 별도의 의사표시가 없을 경우에는 저작물의 이용기간을 계속 연장함.
5. 해당 저작물의 저작권을 타인에게 양도하거나 또는 출판을 허락을 하였을 경우에는 1개월 이내에 대학에 이를 통보함.
6. 조선대학교는 저작물의 이용허락 이후 해당 저작물로 인하여 발생하는 타인에 의한 권리 침해에 대하여 일체의 법적 책임을 지지 않음
7. 소속대학의 협정기관에 저작물의 제공 및 인터넷 등 정보통신망을 이용한 저작물의 전송・출력을 허락함.

2009년 4 월 24 일

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